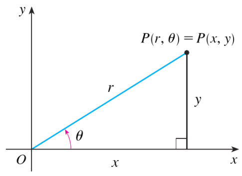
Cálculo III

Coordenadas cilíndricas e esféricas

Prof. Adriano Barbosa

Coordenadas polares



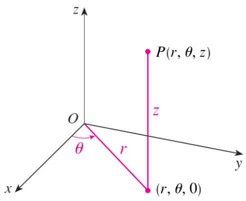
$x = r \cos \theta$

$y = r \operatorname{sen} \theta$

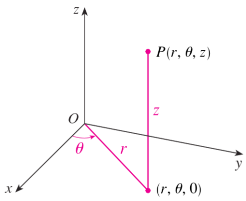
$r^2 = x^2 + y^2$

$\operatorname{tg} \theta = \dfrac{y}{x}$

Coordenadas cilíndricas



Coordenadas cilíndricas



$x = r \cos \theta \qquad y = r \operatorname{sen} \theta \qquad z = z$

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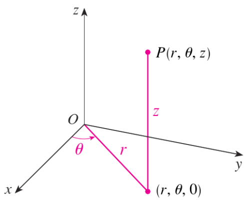
Coordenadas cilíndricas



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Coordenadas cilíndricas são úteis em problemas que envolvem simetria em torno de um eixo e o eixo z é escolhido de modo a coincidir com o eixo de simetria.

Exemplo

Descreva a superfície cuja equação em coordenadas cilíndricas é $r = c$.

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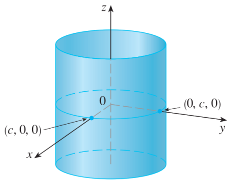

Exemplo

Um sólido E está contido no cilindro $x^2 + y^2 = 1$, abaixo do plano $z = 4$ e acima do paraboloide $z = 1 - x^2 - y^2$.

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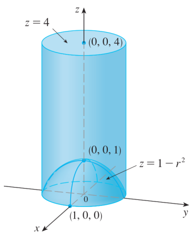

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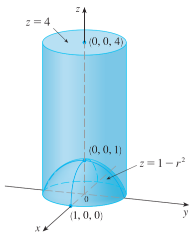
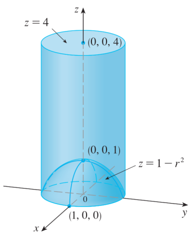


$$\iiint_E K\sqrt{x^2 + y^2}\, dV$$

Exemplo



$$E = \{(r, \theta, z) \mid 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 1,\ 1 - r^2 \leq z \leq 4\}$$

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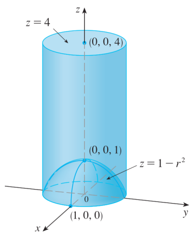

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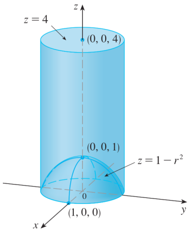
$$\iiint_E K\sqrt{x^2 + y^2}\, dV = \int_0^{2\pi} \int_0^1 \int_{1-r^2}^4 (Kr)\, r\, dz\, dr\, d\theta$$

Exemplo



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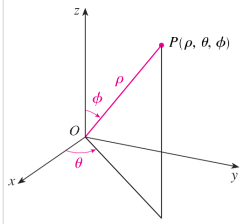
$$= \int_0^{2\pi} \int_0^1 Kr^2[4 - (1 - r^2)]\, dr\, d\theta$$

$$= K \int_0^{2\pi} d\theta \int_0^1 (3r^2 + r^4)\, dr$$

$$= 2\pi K\left[r^3 + \frac{r^5}{5}\right]_0^1 = \frac{12\pi K}{5}$$

Coordenadas esféricas



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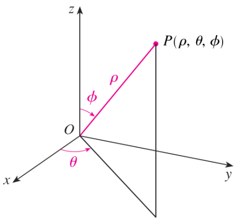


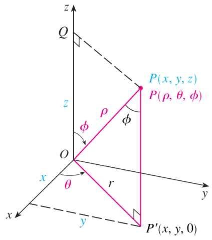
$\rho = |OP|$ é a distância da origem a P

θ é o mesmo ângulo que nas coordenadas cilíndricas

ϕ é o ângulo entre o eixo z positivo e o segmento de reta OP

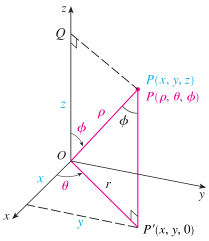
$$\rho \geqslant 0 \qquad 0 \leqslant \phi \leqslant \pi$$

Coordenadas esféricas



$$z = \rho \cos \phi \qquad r = \rho \operatorname{sen} \phi$$

Coordenadas esféricas



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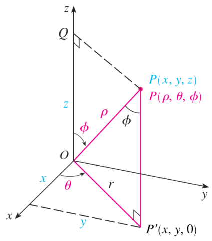

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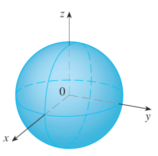
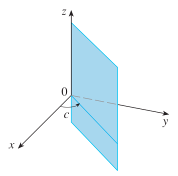
$$x = \rho \operatorname{sen} \phi \cos \theta$$
$$y = \rho \operatorname{sen} \phi \operatorname{sen} \theta$$
$$z = \rho \cos \phi$$
$$\rho^2 = x^2 + y^2 + z^2$$

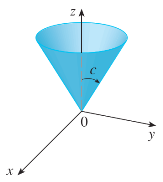
Exemplos



$\rho = c$, uma esfera



$\theta = c$, um semiplano



$0 < c < \pi/2$
$\phi = c$, um cone

Exemplo

Calcule $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$, onde B é a bola unitária:

$$B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\}$$

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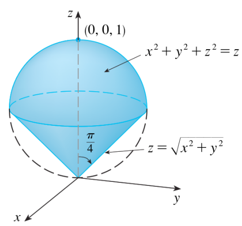
$$\begin{aligned}\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV &= \int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{(\rho^2)^{3/2}} \boxed{\rho^2 \operatorname{sen}\phi}\, d\rho\, d\theta\, d\phi \\ &= \int_0^{\pi} \operatorname{sen}\phi\, d\phi \int_0^{2\pi} d\theta \int_0^1 \rho^2 e^{\rho^3}\, d\rho \\ &= \big[-\cos\phi\big]_0^{\pi}(2\pi)\left[\tfrac{1}{3}e^{\rho^3}\right]_0^1 = \tfrac{4}{3}\pi(e-1)\end{aligned}$$

Exercícios

Utilize coordenadas esféricas para determinar o volume do sólido que fica acima do cone $z = \sqrt{x^2 + y^2}$ e abaixo da esfera $x^2 + y^2 + z^2 = z$.

$\frac{\pi}{8}$



Calcule $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\, dz\, dy\, dx$.

$\frac{16}{5}\pi$